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The value of p in (2) from (1) gives the primitive, which is the curve desired. By series we get

$$\frac{m^2}{2}\int\frac{\sqrt{(p)}dp}{\sqrt{(m^2p^2-1)}}=\frac{1}{2\sqrt{p}}\sqrt{(m^2p^2-1)}+\tfrac{1}{2}m\sqrt{(p)}\left(1+\frac{1}{2.3m^2p^2}+\frac{1}{2.4.7\,m^4p^4}\right.$$

$$\left.+\frac{3}{2.4.6.11\,m^6p^6}+\frac{3.5}{2.4.6.8.15m^8p^8}+\ldots\right).$$

164. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

If $m^2+n^2=1$, $m^2\cos^2\theta+n^2\cos^2\varphi=A$, $a^2b^2\sin^2\theta(m^2+n^2\cos^2\varphi)+a^2c^2\cos^2\theta\cos^2\varphi+b^2c^2\sin^2\varphi(n^2+m^2\cos^2\theta)=B$, $\sqrt{(1-m^2\sin^2\theta)}=\triangle(\theta)$, $\sqrt{(1-n^2\sin^2\varphi)}=\triangle(\varphi)$ prove that $\displaystyle\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\frac{ABd\theta d\varphi}{\triangle(\theta)\triangle(\varphi)}=(\tfrac{1}{6}\pi)(a^2b^2+a^2c^2+b^2c^2)$.

Solution by the PROPOSER.

$x^2/a^2+y^2/b^2+z^2/c^2=1$.

$$\int ds=\int\int\sqrt{[1+(dz/dx)^2+(dz/dy)^2]}dxdy=c^2\int\int\frac{\sqrt{[z^2/c^4+x^2/a^4+y^2/b^4]}dxdy}{z}.$$

Let p=perpendicular from center on tangent plane.

$\therefore\ 1/p=\sqrt{[z^2/c^4+x^2/a^4+y^2/b^4]}$.

Let $(b/a)\sqrt{[a^2-x^2]}=y'$, $(a/b)\sqrt{[b^2-y^2]}=x'$.

$$D=\int\frac{ds}{p}=c^2\int\int\frac{[z^2/c^4+x^2/a^4+y^2/b^4]dxdy}{z}=\frac{1}{c^2}\int_0^a\int_0^{y'}zdxdy+\frac{c^2}{a^4}\int_0^a\int_0^{y'}\frac{x^2}{z}dxdy$$

$$+\frac{c^2}{b^4}\int_0^b\int_0^{x'}(y^2/z)dydx=\frac{1}{abc}\int_0^a\int_0^{y'}\sqrt{[a^2b^2-b^2x^2-a^2y^2]}dxdy$$

$$+\frac{abc}{a^4}\int_0^a\int_0^{y'}\frac{x^2dxdy}{\sqrt{[a^2b^2-b^2x^2-a^2y^2]}}+\frac{abc}{b^4}\int_0^b\int_0^{x'}\frac{y^2dydx}{\sqrt{[a^2b^2-b^2x^2-a^2y^2]}}$$

$$=\tfrac{1}{6}\pi(ab/c+bc/a+ac/b)=\frac{\pi}{6abc}(a^2b^2+b^2c^2+a^2c^2).$$

Let $x=a\sin\varphi\sqrt{(1-m^2\sin^2\theta)}$, $y=b\cos\theta\cos\varphi$, $z=c\sin\theta\sqrt{(1-n^2\sin^2\varphi)}$, $m^2+n^2=1$; $dx/d\varphi=a\cos\varphi\sqrt{(1-m^2\sin^2\theta)}$, $dx/d\theta=-\dfrac{am^2\sin\varphi\sin\theta\cos\theta}{\sqrt{(1-m^2\sin^2\theta)}}$, $dy/d\varphi=-b\cos\theta\sin\varphi$, $dy/d\theta=-b\sin\theta\cos\varphi$; $dxdy=(dx/d\theta)(dy/d\varphi)-(dx/d\varphi)(dy/d\theta)$

$$=\frac{ab}{\sqrt{(1-m^2\sin^2\theta)}}(m^2\sin^2\varphi\sin\theta\cos^2\theta+\sin\theta\cos^2\varphi-m^2\sin^3\theta\cos^2\varphi)=\frac{Aab\sin\theta}{\Delta(\theta)}.$$

$$\frac{c^2}{z}(x^2/a^4+y^2/b^4+z^2/c^4)=\frac{c}{\sin\theta\,\Delta(\varphi)}\Big(\frac{\sin^2\varphi(1-m^2\sin^2\theta)}{a^2}+\frac{\cos^2\theta\cos^2\varphi}{b^2}$$

$$+\frac{\sin^2\theta(1-n^2\sin^2\varphi)}{c^2}\Big)=\frac{1}{a^2b^2c\sin\theta\,\Delta(\varphi)}[b^2c^2\sin^2\varphi(n^2+m^2\cos^2\theta)$$

$$+a^2c^2\cos^2\theta\cos^2\varphi+a^2b^2\sin^2\theta(m^2+n^2\cos^2\varphi)]=\frac{B}{a^2b^2c\sin\theta\,\Delta(\varphi)}.$$

$$\therefore\int\frac{ds}{p}=\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\frac{ABd\theta d\varphi}{abc\Delta(\theta)\Delta(\varphi)}=\frac{\pi}{6abc}(a^2b^2+a^2c^2+b^2c^2).$$

$$\therefore\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\frac{ABd\theta d\varphi}{\Delta(\theta)\Delta(\varphi)}=\tfrac{1}{6}\pi(a^2b^2+a^2c^2+b^2c^2).$$

Also $\displaystyle\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\frac{ABd\theta d\varphi}{\Delta(\theta)\Delta(\varphi)}=\tfrac{1}{3}(a^2b^2+a^2c^2+b^2c^2)[E(m)F(n)$

$+F(m)E(n)-F(m)F(n)]$.

$\therefore\ E(m)F(n)+F(m)E(n)-F(m)F(n)=\frac{1}{2}\pi$, Legendre's Theorem.

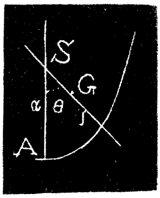
MECHANICS.

155. Proposed by M. E. GRABER. Graduate Student, Heidelberg University, Tiffin, Ohio.

A parabolic curve is placed in a vertical plane with its axis vertical and vertex downwards, and inside of it, and against a peg in the focus, and against the concave arc, a smooth uniform and heavy beam rests. Find the position of equilibrium.

Solution by G. W. GREENWOOD, A. B., Professor of Mathematics, McKendree College, Lebanon, Ill., and E. L. SHERWOOD, Shady Side Academy, Pittsburg, Pa.

Let the equation to the parabola be $2a/r=1+\cos\theta$, and let the length of the rod be $2l$. We must have $2l>a$. When the rod makes an angle θ with the axis, the depth of its center of gravity below the focus is



$$n=(r-l)\cos\theta=\Big(\frac{2a}{1+\cos\theta}-l\Big)\cos\theta.$$

In a position of equilibrium, n is a maximum or a minimum.

$$\therefore\ \sin\theta\Big(\frac{2a}{(1+\cos\theta)^2}-l\Big)=0\quad\therefore\ \theta=0,\ \text{or}\ \cos\theta=\sqrt{(2a/l)}-1.$$